Análise Fatorial Confirmatória CFA

Regressão, Modelos de Traço Latente, Modelagem com Equações Estruturais

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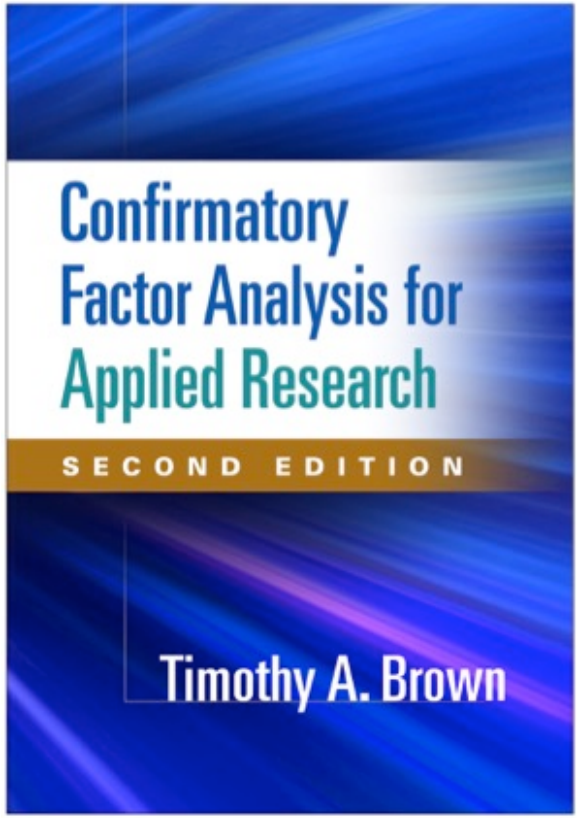
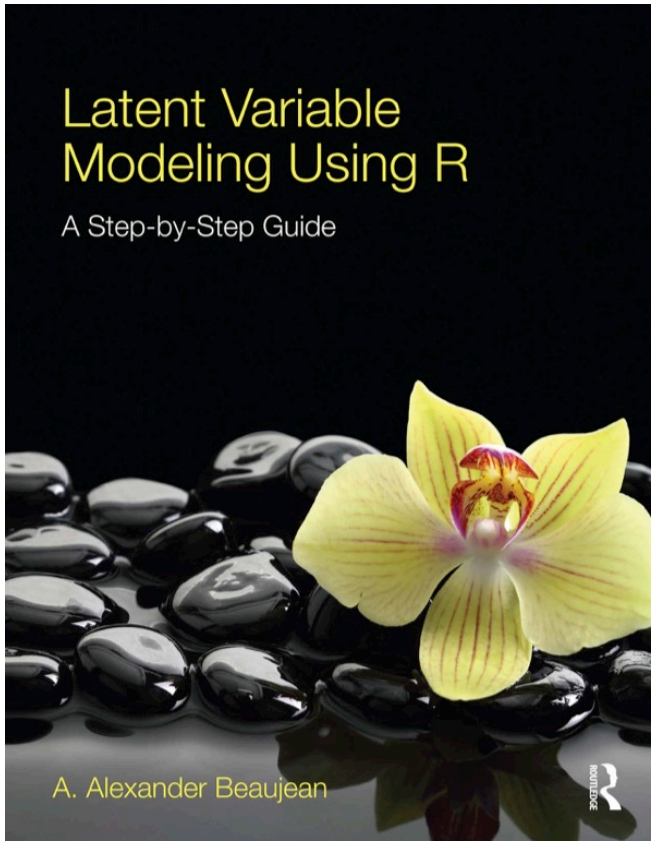
2018

- http://blogs.baylor.edu/rlatentvariable/

Beaujean cap 3
Brown cap. 2 e 3





3.1 Background

A **structural equation model** (SEM) is a broad class of statistical models (see Figure 3.1) that consists of two parts: the structural model and the latent variable model. The structural model consists of the regression-like relationships among the variables (i.e., the path analysis from Chapter 2). The **latent variable model** (LVM) forms the latent variables (LVs) used in the structural model. When a LVM is analyzed without a structural model, it is sometimes called a **confirmatory factor analysis** (CFA). If there was not a hypothesized structure for the latent variable model, then it would be an **exploratory factor analysis** (EFA). The majority of this book is focused on confirmatory LVMs, although in Section 3.4 I demonstrate fitting a full SEM. Beaujean (2013) demonstrates how to conduct an EFA in **R**.

Structural Equation Model (SEM)
Latent Variable Model (LVM)
Confirmatory Factor Analysis (CFA)
Exploratory Factor Analysis (EFA)

The *factor* in factor analysis is synonymous with a latent variable.

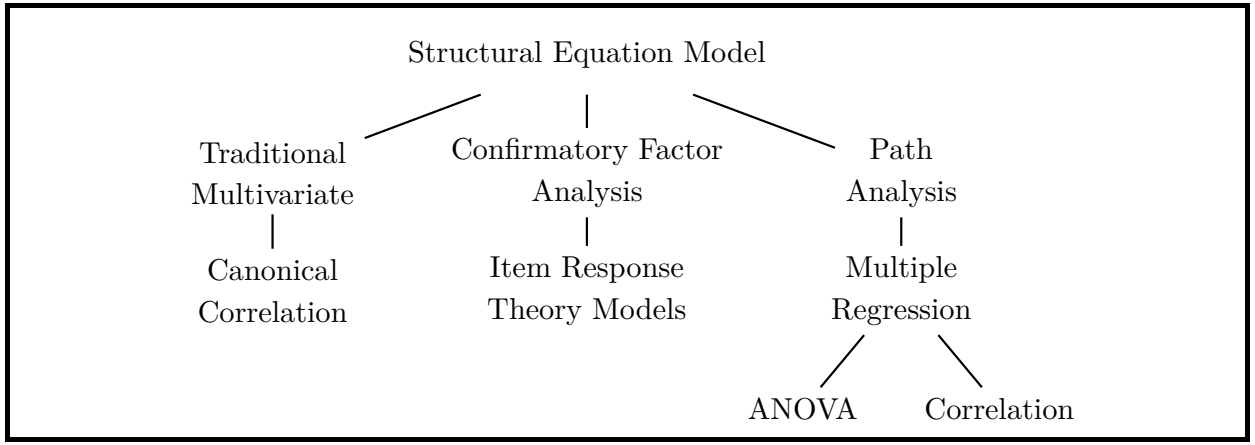
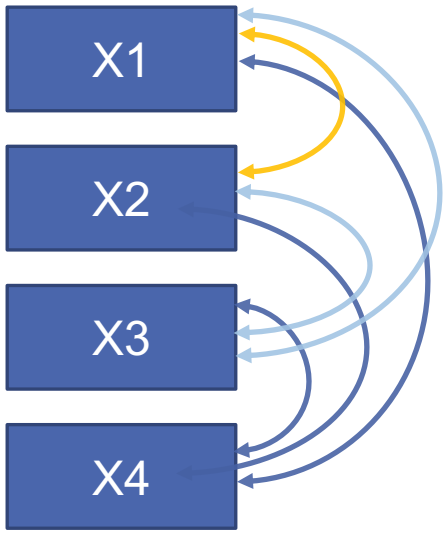


Figure 3.1 Example of data analysis methods subsumed by a structural equation model.

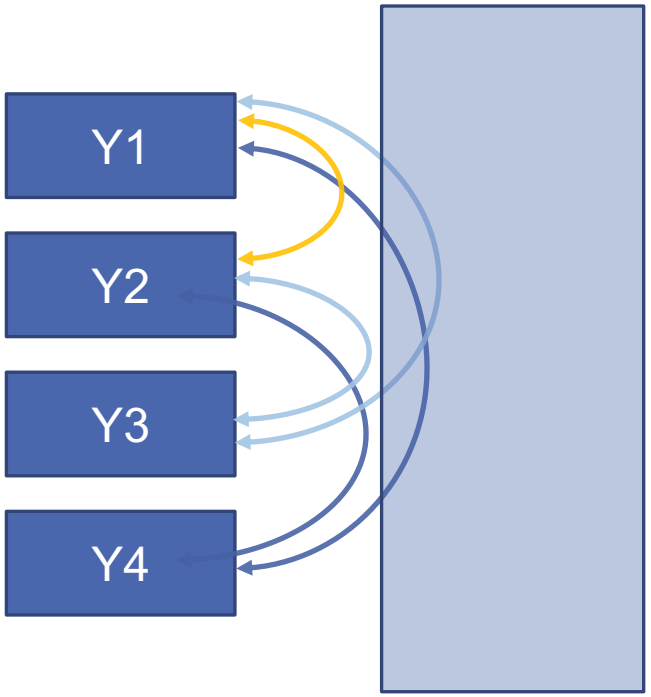
Informação não redundante= $\frac{p(p+1)}{2} = \frac{4(4+1)}{2}$=10

6 correlações e 4 variâncias

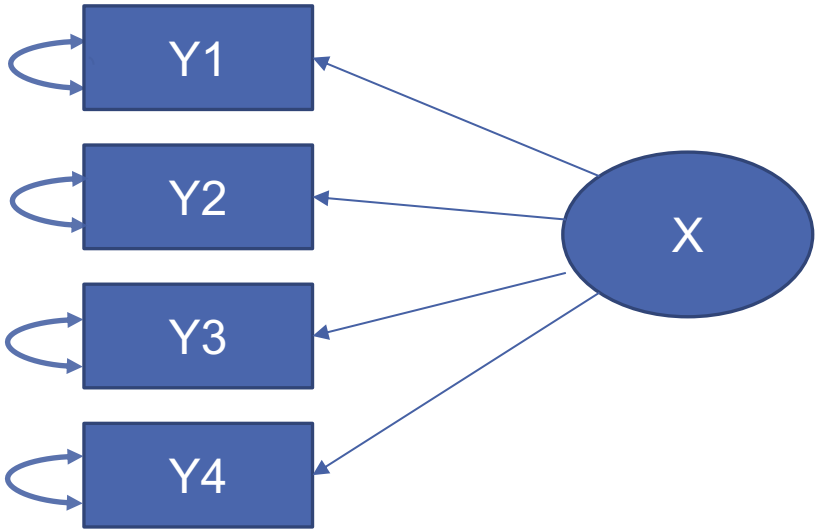


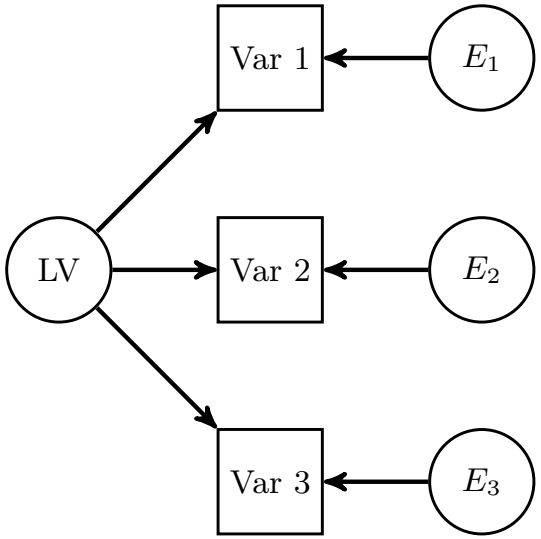
Informação não redundante= $\frac{p(p+1)}{2} = \frac{4(4+1)}{2}$=10

6 correlações e 4 variâncias

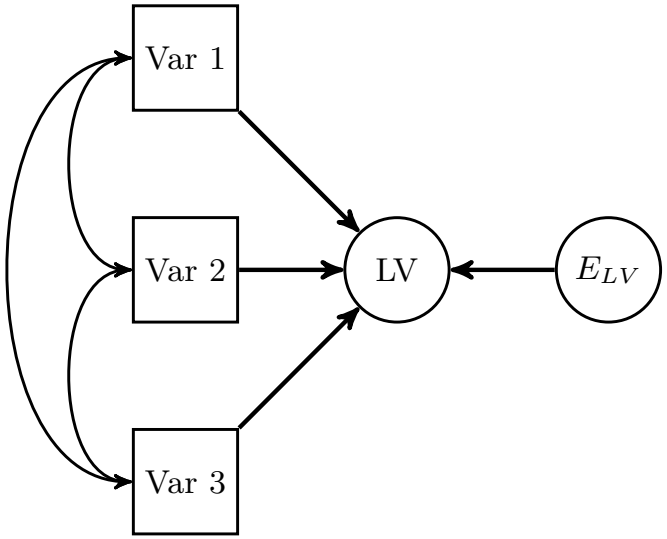


4 cargas fatoriais e 4 variâncias de erro





(a) *LV* is a reflective latent variable.



(b) *LV* is a formative latent variable.

Figure 3.2 Reflective and formative latent variable path models.

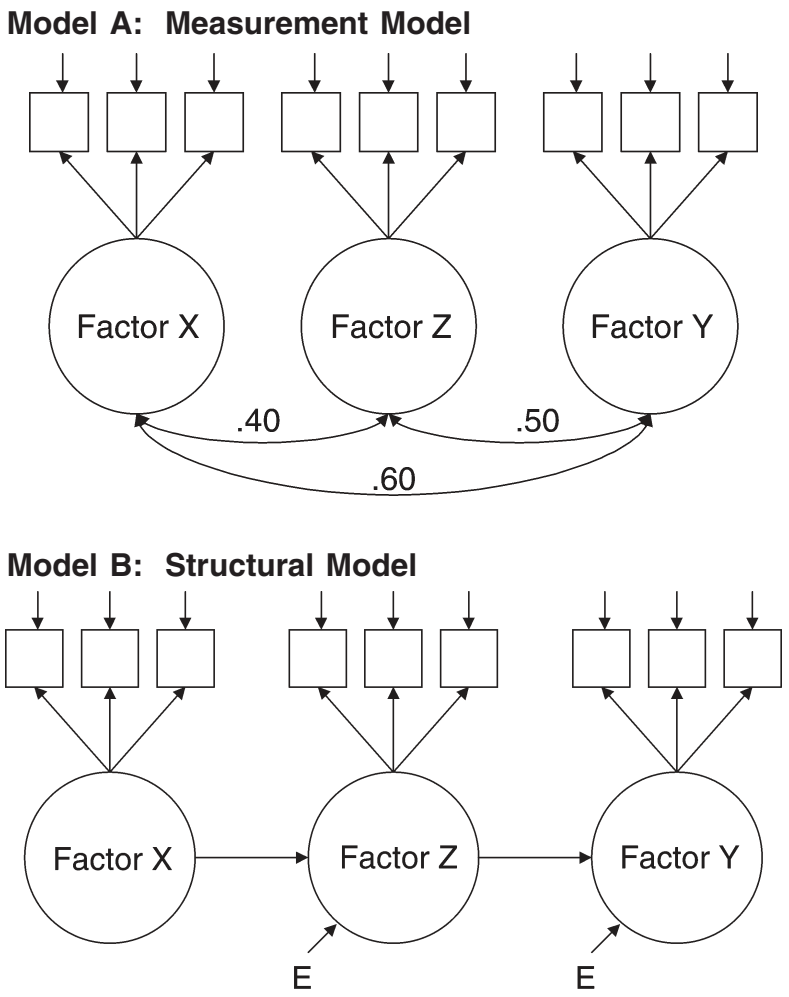


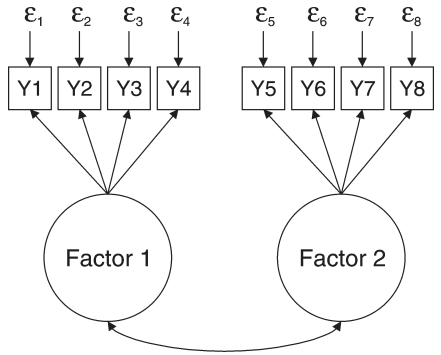
FIGURE 3.2. Path diagrams of measurement and structural models.

Conceitos importantes

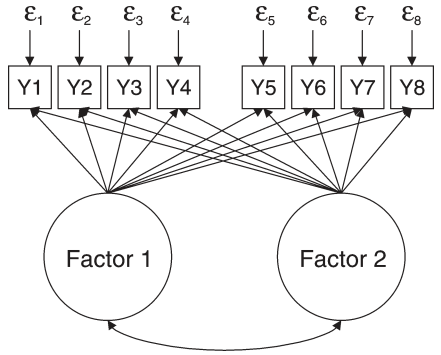
- EFA/ CFA Cargas fatoriais e comunalidade
  - Pattern coefficients (*corr parciais*)
  - Structural coefficients (*corr zero order*)
  - Comunalidade e unicidade ($\boldsymbol{R}^2$)
- Identificação
  - GI = info – parâmetros
  - Just-, under e over identified
- Número de indicadores
- Métrica da variável latente
  - Padronizada (0/1), variável marcadora e effect coding

EFA vs CFA

Model A: Confirmatory Factor Model (all measurement error is random)



Model B: Exploratory Factor Model (oblique rotation)



Model C: Confirmatory Factor Model (with a correlated measurement error)

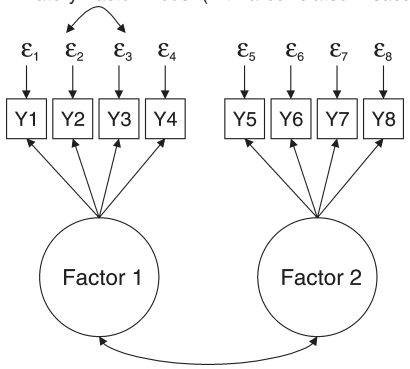


FIGURE 3.1. Path diagrams of confirmatory and exploratory factor models.

TABLE 3.1. Factor Loading Matrices from EFA and CFA of Adolescent Antisocial Behaviors

A. CFA (factor correlation = .6224)

	Factor		
	Property Crimes	Violent Crimes	Communality
Y1	.7996	.0000	.64
Y2	.6451	.0000	.42
Y3	.5699	.0000	.32
Y4	.4753	.0000	.23
Y5	.0000	.7315	.53
Y6	.0000	.5891	.35
Y7	.0000	.7446	.55
Y8	.0000	.5803	.34

B. EFA (oblique rotation, factor correlation = .5722)

	Factor		
	Property Crimes	Violent Crimes	Communality
Y1	.9187	–.0958	.75
Y2	.5422	.1045	.37
Y3	.5300	.0372	.30
Y4	.4494	.0103	.21
Y5	.0434	.7043	.53
Y6	–.1178	.6999	.41
Y7	.1727	.6106	.52
Y8	.0264	.5756	.35

C. EFA (orthogonal rotation, factor correlation = 0)

	Factor		
	Property Crimes	Violent Crimes	Communality
Y1	.8493	.1765	.75
Y2	.5509	.2574	.37
Y3	.5185	.1898	.30
Y4	.4331	.1408	.21
Y5	.2587	.6826	.53
Y6	.1032	.6314	.41
Y7	.3535	.6312	.52
Y8	.2028	.5552	.35

Note. N = 1,050. Y1 = shoplifting, Y2 = vandalism, Y3 = theft, Y4 = broke into building/vehicle, Y5 = fighting, Y6 = aggravated assault, Y7 = hit family/teachers, Y8 = threatened others.

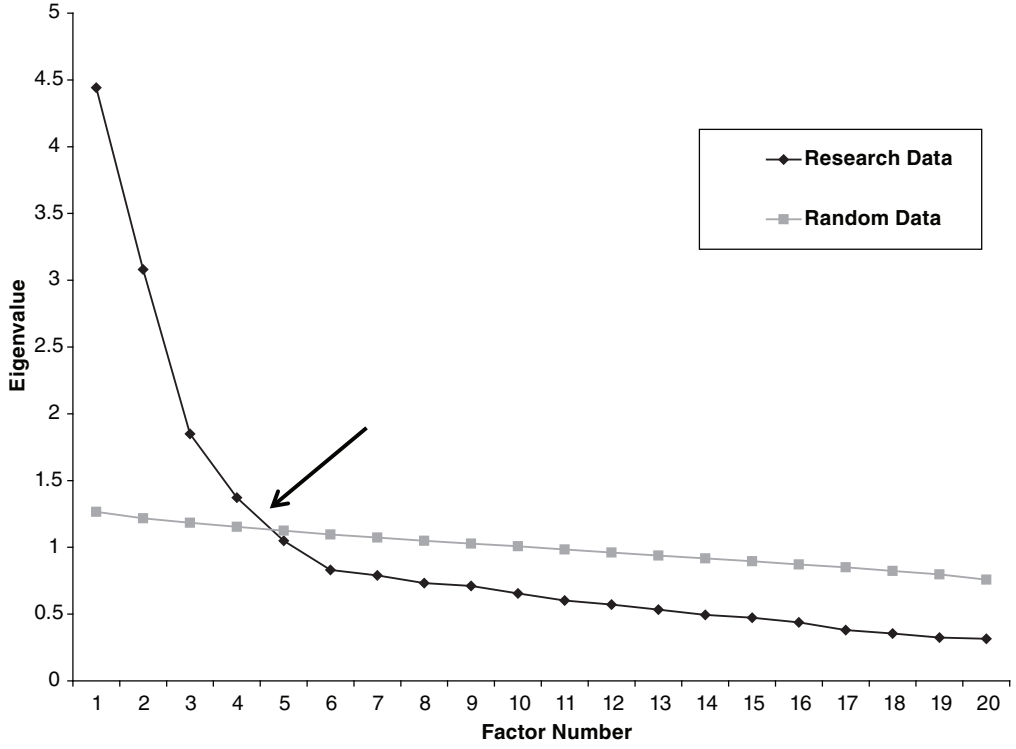
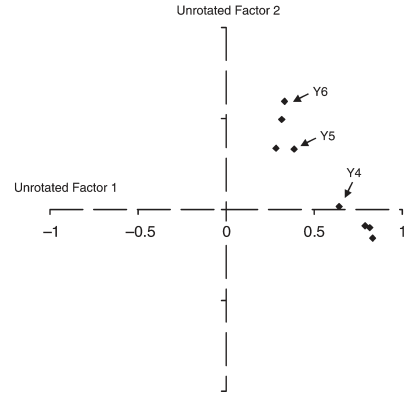


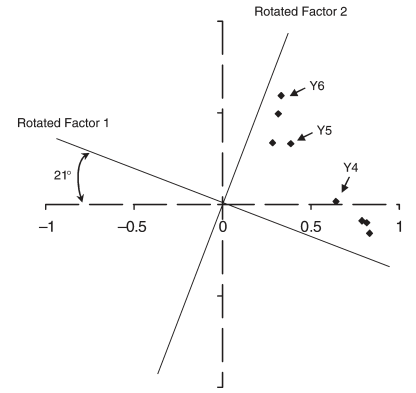
FIGURE 2.3. Parallel analysis using eigenvalues from research and random data (average of 50 replications). Arrow indicates that eigenvalues from random data exceed the eigenvalues from research data after the fourth factor.

A. Unrotated Factor Matrix



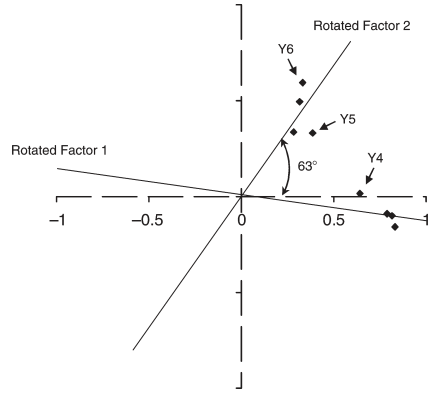
	Factor	
	1	2
Y1	.834	–.160
Y2	.813	–.099
Y3	.788	–.088
Y4	.642	.015
Y5	.386	.329
Y6	.333	.593
Y7	.313	.497
Y8	.284	.336

B. Orthogonally Rotated Factor Matrix (Varimax)



	Factor	
	1	2
Y1	.836	.150
Y2	.794	.199
Y3	.767	.201
Y4	.594	.244
Y5	.242	.445
Y6	.098	.673
Y7	.114	.576
Y8	.145	.416

C. Obliquely Rotated Factor Matrix (Promax)



	Factor	
	1	2
Y1	.875	–.062
Y2	.817	.003
Y3	.788	.012
Y4	.588	.106
Y5	.154	.418
Y6	–.059	.704
Y7	–.018	.595
Y8	.055	.413

FIGURE 2.4. Geometric representations of unrotated, orthogonally rotated, and obliquely rotated factor matrices.

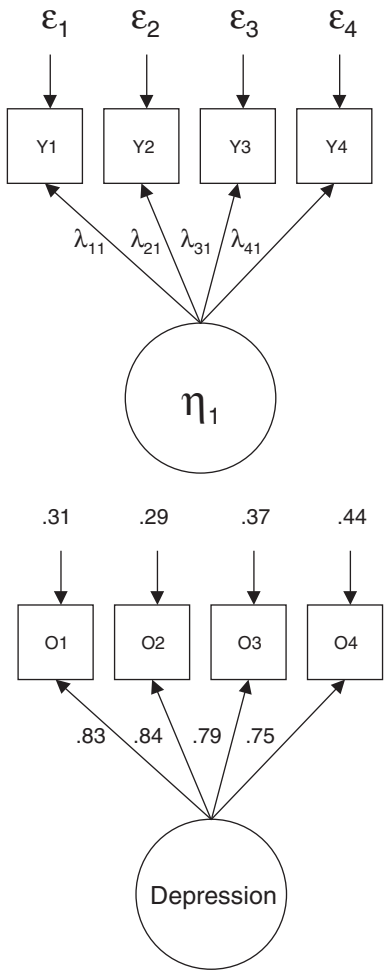


FIGURE 2.1. Path diagram of the one-factor model.

A fundamental equation of the common factor model is

$$y_j = \lambda_j 1\eta_1 + \lambda_j 2\eta_2 + \ldots + \lambda_{jm}\eta_m + \varepsilon_j \tag{2.1}$$

where y_j represents the jth of p indicators (in the case $p = 4$; O1, O2, O3, O4) obtained from a sample of n independent participants (in this case, $n = 300$); λ_{jm} represents the factor loading relating variable j to the mth factor η (in the case $m = 1$; the single factor of Depression); and ε_j represents the variance that is unique to indicator y_j and is independent of all ηs and all other εs. As will be seen in subsequent chapters, similar notation is used to represent some of the equations of CFA. In this simple factor solution entailing a single factor (η_1) and four indicators, the regression functions depicted in Figure 2.1 can be summarized by four separate equations:

$$\begin{aligned} O1 &= \lambda_{11}\eta_1 + \varepsilon_1 \\ O2 &= \lambda_{21}\eta_1 + \varepsilon_2 \\ O3 &= \lambda_{31}\eta_1 + \varepsilon_3 \\ O4 &= \lambda_{41}\eta_1 + \varepsilon_4 \end{aligned} \tag{2.2}$$

This set of equations can be summarized in a single equation that expresses the relationships among observed variables (y), factors (η), and unique variances (ε):

$$y = \Lambda_y\eta + \varepsilon \tag{2.3}$$

or in expanded matrix form:

$$\Sigma = \Lambda_y\Psi\Lambda'_y + \Theta\varepsilon \tag{2.4}$$

$$\begin{aligned} \text{VAR}(O1) = \sigma_{11} &= \lambda_{11}{}^2\psi_{11} + \varepsilon_1 \\ &= .828^2(1) + .315 \\ &= 1.00 \end{aligned} \tag{2.5}$$

$$\begin{aligned} \text{COV}(O1, O2) = \sigma_{21} &= \lambda_{11}\psi_{11}\lambda_{21} \\ &= (.828)(1)(.841) \\ &= .696 \end{aligned} \tag{2.6}$$

CFA

CFA

- Modelos de medida. Ancorados na teoria
  - Validade? Até que ponto os indicadores são medidas válidas do construto ?
  - Efeitos da variável latente?
  - Até que ponto a variável latente é construto que se presume ?
- Recurso para testar a dimensionalidade das medidas
- Recurso para testar modelos alternativos
  - Especifica-se dois modelos e verifica-se qual deles se ajusta melhor aos dados
- Recurso para estimar correlações entre construtos não-atenuadas pelos erros de medida

GREEK ALPHABET

By Ben Crowder • bencrowder.net • Last modified 2 May 2012

Αα	Ββ	Γγ	Δδ	Εε	Ζζ
ALPHA [a]	**BETA** [b]	**GAMMA** [g]	**DELTA** [d]	**EPSILON** [e]	**ZETA** [dz]
ἄλφα	*βῆτα*	*γάμμα*	*δέλτα*	*ἒ ψιλόν*	*ζῆτα*
Ηη	Θθ	Ιι	Κκ	Λλ	Μμ
ETA [ɛː]	**THETA** [tʰ]	**IOTA** [i]	**KAPPA** [k]	**LAMBDA** [l]	**MU** [m]
ἦτα	*θῆτα*	*ἰῶτα*	*κάππα*	*λάμβδα*	*μῦ*
Νν	Ξξ	Οο	Ππ	Ρρ	Σσς
NU [n]	**XI** [ks]	**OMICRON** [o]	**PI** [p]	**RHO** [r]	**SIGMA** [s]
νῦ	*ξεῖ*	*ὀ μικρόν*	*πεῖ*	*ῥῶ*	*σῖγμα*
Ττ	Υυ	Φφ	Χχ	Ψψ	Ωω
TAU [t]	**UPSILON** [ʉ]	**PHI** [pʰ]	**CHI** [kʰ]	**PSI** [ps]	**OMEGA** [ɔː]
ταῦ	*ὖ ψιλόν*	*φεῖ*	*χεῖ*	*ψεῖ*	*ὦ μέγα*

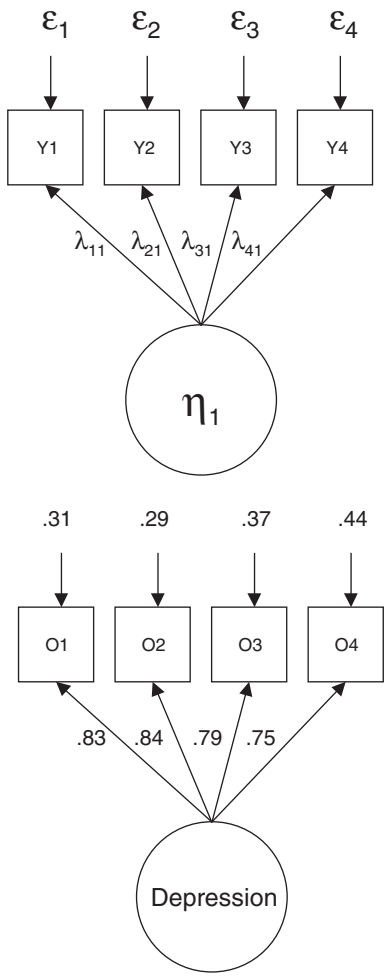
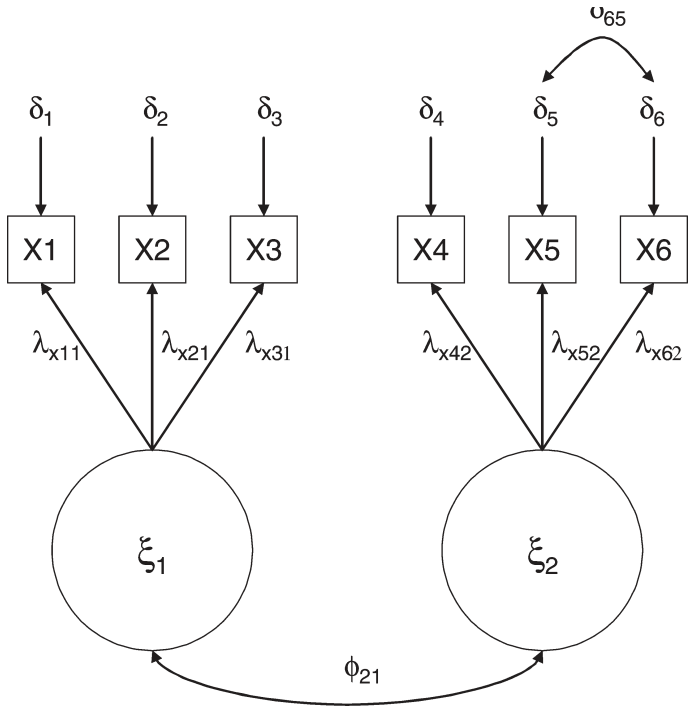
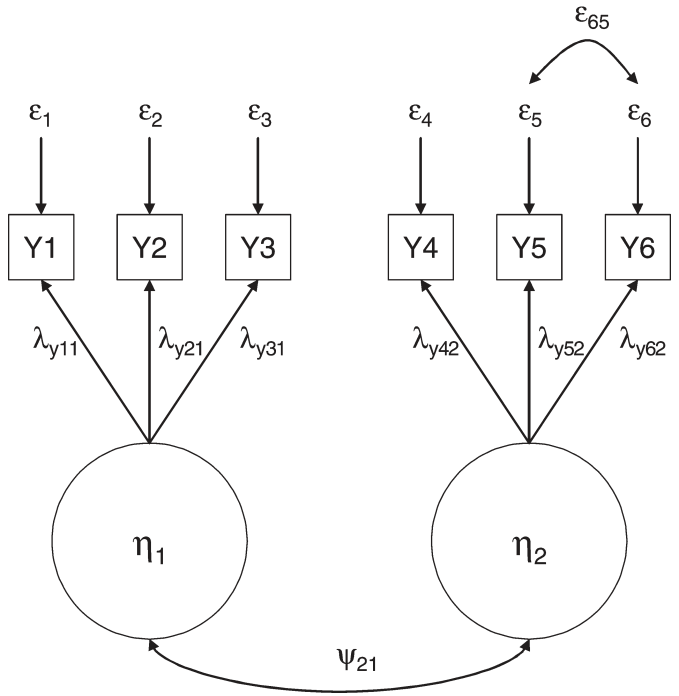


FIGURE 2.1. Path diagram of the one-factor model.



Name	Parameter	Matrix	Type	Description
Lambda-X	λ_x	Λ_x	Regression	Factor loadings
Theta-delta	δ	Θ_δ	Variance–covariance	Error variances and covariances
Phi	ϕ	Φ	Variance–covariance	Factor variances and covariances
Tau-X	τ_x		Mean vector	Indicator intercepts
Kappa	κ		Mean vector	Latent means
Xi (Ksi)	ξ		Vector	Names of exogenous variables

FIGURE 3.3. Latent X notation for a two-factor CFA model with one error covariance. Factor variances, factor means, and indicator intercepts are not depicted in the path diagram.



Name	Parameter	Matrix	Type	Description
Lambda-Y	λ_y	Λ_y	Regression	Factor loadings
Theta-epsilon	ε	Θ_ε	Variance–covariance	Error variances and covariances
Psi	ψ	Ψ	Variance–covariance	Factor variances and covariances
Tau-Y	τ_y		Mean vector	Indicator intercepts
Alpha	α		Mean vector	Latent means
Eta	η		Vector	Names of endogenous variables

(3.1)

	ξ_1	ξ_2
X1	λ_{x11}	0
X2	λ_{x21}	0
X3	λ_{x31}	0
X4	0	λ_{x42}
X5	0	λ_{x52}
X6	0	λ_{x62}

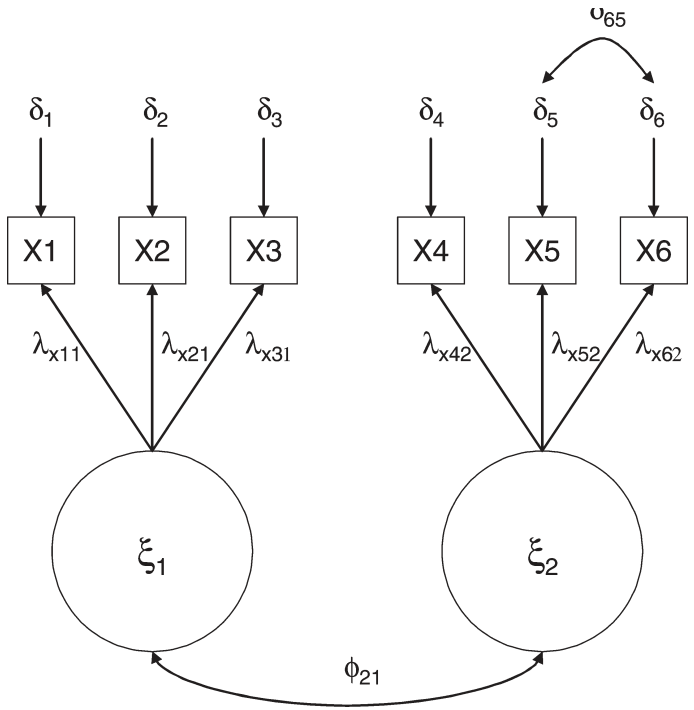
(3.2)

	ξ_1	ξ_2
ξ_1	ϕ_{11}	
ξ_2	ϕ_{21}	ϕ_{22}

(3.3)

	X1	X2	X3	X4	X5	X6
X1	δ_{11}					
X2	0	δ_{22}				
X3	0	0	δ_{33}			
X4	0	0	0	δ_{44}		
X5	0	0	0	0	δ_{55}	
X6	0	0	0	0	δ_{65}	δ_{66}

FIGURE 3.4. Latent Y notation for a two-factor CFA model with one error covariance. Factor variances, factor means, and indicator intercepts are not depicted in the path diagram.



Name	Parameter	Matrix	Type	Description
Lambda-X	λ_x	Λ_x	Regression	Factor loadings
Theta-delta	δ	Θ_δ	Variance–covariance	Error variances and covariances
Phi	ϕ	Φ	Variance–covariance	Factor variances and covariances
Tau-X	τ_x		Mean vector	Indicator intercepts
Kappa	κ		Mean vector	Latent means
Xi (Ksi)	ξ		Vector	Names of exogenous variables

$$\begin{array}{lcc} & \xi_1 & \xi_2 \\ X1 & \lambda_{x11} & 0 \\ X2 & \lambda_{x21} & 0 \\ X3 & \lambda_{x31} & 0 \\ X4 & 0 & \lambda_{x42} \\ X5 & 0 & \lambda_{x52} \\ X6 & 0 & \lambda_{x62} \end{array} \qquad (3.1)$$

$$\begin{array}{lcc} & \xi_1 & \xi_2 \\ \xi_1 & \phi_{11} & \\ \xi_2 & \phi_{21} & \phi_{22} \end{array} \qquad (3.2)$$

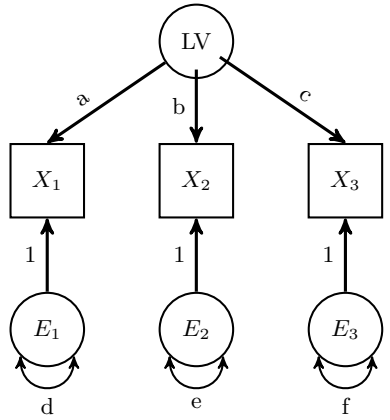
$$\begin{array}{lcccccc} & X1 & X2 & X3 & X4 & X5 & X6 \\ X1 & \delta_{11} & & & & & \\ X2 & 0 & \delta_{22} & & & & \\ X3 & 0 & 0 & \delta_{33} & & & \\ X4 & 0 & 0 & 0 & \delta_{44} & & \\ X5 & 0 & 0 & 0 & 0 & \delta_{55} & \\ X6 & 0 & 0 & 0 & 0 & \delta_{65} & \delta_{66} \end{array} \qquad (3.3)$$

FIGURE 3.3. Latent X notation for a two-factor CFA model with one error covariance. Factor variances, factor means, and indicator intercepts are not depicted in the path diagram.

Identificação e definição da métrica

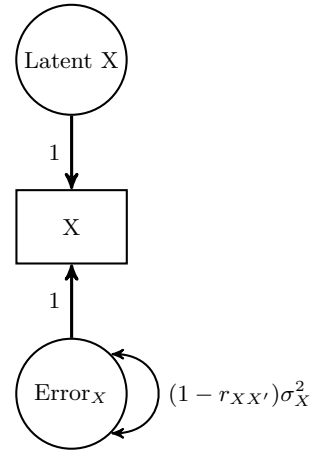
	X_1	X_2	X_3
X_1	σ_1^2		
X_2	σ_{12}	σ_2^2	
X_3	σ_{13}	σ_{23}	σ_3^2

(a) Covariance matrix showing non-redundant elements.



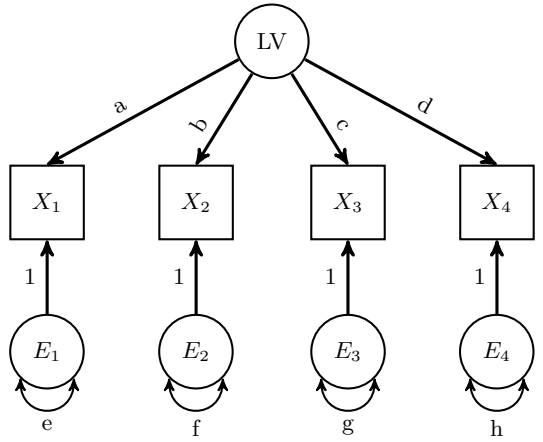
(b) Latent variable model.

Figure 3.4 Example with three indicator variables.



	X_1	X_2	X_3	X_4
X_1	σ_1^2			
X_2	σ_{12}	σ_2^2		
X_3	σ_{13}	σ_{23}	σ_3^2	
X_4	σ_{14}	σ_{24}	σ_{34}	σ_4^2

(a) Covariance matrix showing non-redundant elements.



(b) Latent variable model.

Figure 3.5 Example with four indicator variables.

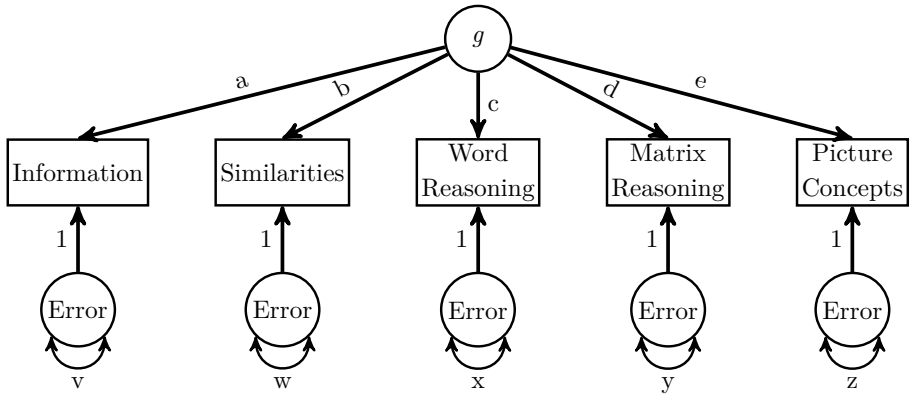
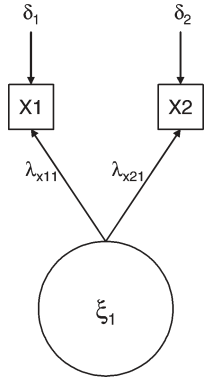


Figure 3.3 Single-factor model of five Wechsler Intelligence Scale for Children-Fourth Edition subtests.

$$n = 550$$

$$df = \underbrace{(5 \times 6/2)}_{\substack{\text{non-redundant} \\ \text{information}}} - \underbrace{(4+5)}_{\substack{\text{loadings +} \\ \text{error varainces}}} - \underbrace{(1)}_{\substack{\text{latent} \\ \text{variances}}} = 15 - 9 - 1 = 5$$

Model A: Underidentified (*df* = –1)

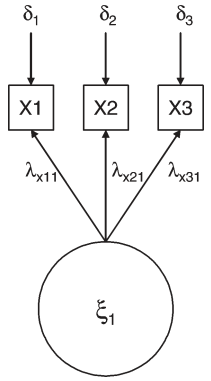


Input Matrix (3 elements)

	X1	X2
X1	σ_{11}	
X2	σ_{21}	σ_{22}

Freely Estimated Model Parameters = 4
(e.g., 2 factor loadings, 2 error variances)

Model B: Just-Identified (*df* = 0)

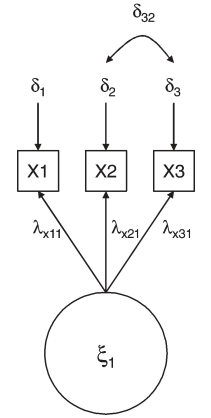


Input Matrix (6 elements)

	X1	X2	X3
X1	σ_{11}		
X2	σ_{21}	σ_{22}	
X3	σ_{31}	σ_{32}	σ_{33}

Freely Estimated Model Parameters = 6
(e.g., 3 factor loadings, 3 error variances)

Model C: Underidentified (*df* = –1)

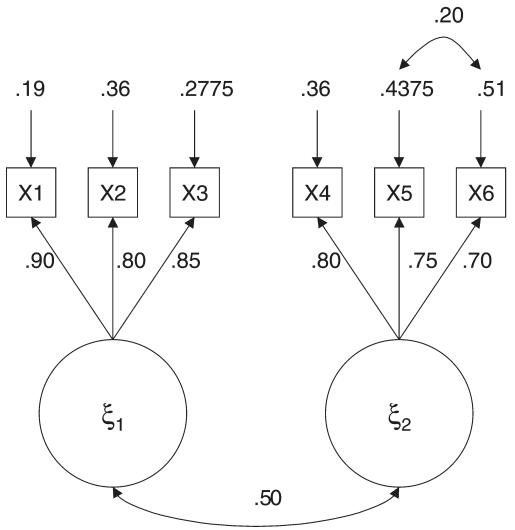


Input Matrix (6 elements)

	X1	X2	X3
X1	σ_{11}		
X2	σ_{21}	σ_{22}	
X3	σ_{31}	σ_{32}	σ_{33}

Freely Estimated Model Parameters = 7
(e.g., 3 factor loadings, 3 error variances, 1 error covariance)

FIGURE 3.6. Examples of underidentified and just-identified CFA models.



Predicted Variances–Covariances (Correlations):

	X1	X2	X3	X4	X5	X6
X1	1.00000					
X2	0.72000	1.00000				
X3	0.76500	0.68000	1.00000			
X4	0.36000	0.32000	0.34000	1.00000		
X5	0.33750	0.30000	0.31875	0.60000	1.00000	
X6	0.31500	0.28000	0.29750	0.56000	0.72500	1.00000

FIGURE 3.5. Reproduction of the input matrix from the parameter estimates of a two-factor measurement model (completely standardized solution).

$$\text{VAR}(X2) = \sigma_{22} = \lambda_{x21}^2\phi_{11} + \delta_2 \quad (3.4)$$
$$= .80^2(1) + .36$$
$$= 1.00$$

$$\eta^2_{22} = \lambda_{x21}^2 \quad (3.5)$$
$$= .80^2$$
$$= .64$$

$$\delta_2 = 1 - \lambda_{x21}^2 \quad (3.6)$$
$$= 1 - .80^2$$
$$= .36$$

$$\text{COV}(X2, X3) = \sigma_{3,2} = \lambda_{x21}\phi_{11}\lambda_{x31} \quad (3.7)$$
$$= (.80)(1)(.85)$$
$$= .68$$

$$\text{COV}(X3, X4) = \sigma_{4,3} = \lambda_{x31}\phi_{21}\lambda_{x42} \quad (3.8)$$
$$= (.85)(.50)(.80)$$
$$= .34$$

$$\text{COV}(X5, X6) = \sigma_{6,5} = (\lambda_{x52}\phi_{22}\lambda_{x62}) + \delta_{65} \quad (3.9)$$
$$= (.75)(1)(.70) + .20$$
$$= .725$$

In the first and by far the more popular method, the researcher fixes the metric of the latent variable to be the same as one of its indicators. The indicator selected to pass its metric on to the factor is often referred to as a *marker* or *reference indicator.* The guidelines for selecting and specifying marker indicators are discussed in Chapter 4. When a marker indicator is specified, a portion of its sample variance is passed on to the latent variable. Using Figure 3.5, suppose X1 is selected as the marker indicator for ξ_1 and has a sample variance (σ_{11}) of 16. Because X1 has a completely standardized factor loading on ξ_1 of .90, 81% of its variance is explained by ξ_1; $.90^2 = .81$ (cf. Eq. 3.5). Accordingly, 81% of the sample variance in X1 is passed on to ξ_1 to represent the factor variance of ξ_1:

$$\begin{aligned} \phi_{11} &= \lambda_{x11}{}^2\sigma_{11} \\ &= (.81)16 \\ &= 12.96 \end{aligned} \tag{3.10}$$

Exercício 5

- http://www.labape.com.br/rprimi/SEM/exerc18/Ex5.html

```
# marker variable
wisc4.model.Std<-'
g =~ NA*Information + a*Information + b*Similarities + c*Word.Reasoning +
d*Matrix.Reasoning + e*Picture.Concepts

# constrain the LV variance to 1
g~~1*g
'
wisc4.fit.Std <- cfa(wisc4.model.Std, sample.cov=wisc4.cor, sample.nobs=550)
# equivalent model
wisc4.fit.Std <- cfa(wisc4.model, sample.cov=wisc4.cor, sample.nobs=550, std.lv=TRUE)
```

3.3.2 Example: Latent Variable Model with Two Latent Variables

Lets say the model in Figure 3.3 is wrong; it should have had two LVs instead of one. Specifically, g should be replaced with LVs representing Verbal Comprehension and Fluid Reasoning, as is shown in Figure 3.9. The steps in estimating this model are very similar to the model with one LV, except I have to specify a slightly different model.

```
# two-factor model of the WISC-IV data
wisc4.model2<-'
V =~ a*Information + b*Similarities + c*Word.Reasoning
F =~ d*Matrix.Reasoning + e*Picture.Concepts
V~~f*F
'
wisc4.fit2 <- cfa(wisc4.model2, sample.cov=wisc4.cov, sample.nobs=550)
```

The results are given in Table 3.3. The table includes both loadings (pattern coefficients) and structure coefficients (see Section 3.2). To calculate the structure coefficients, I used the tracing rules to trace the paths from a MV to a given LV. For example the structure coefficient for the Information subtest and the Fluid LV is: $af = (0.86)(0.82) = 0.71$.

[3]The less than, `<`, and greater than, `>`, logical operators can be used in `lavaan` as well.

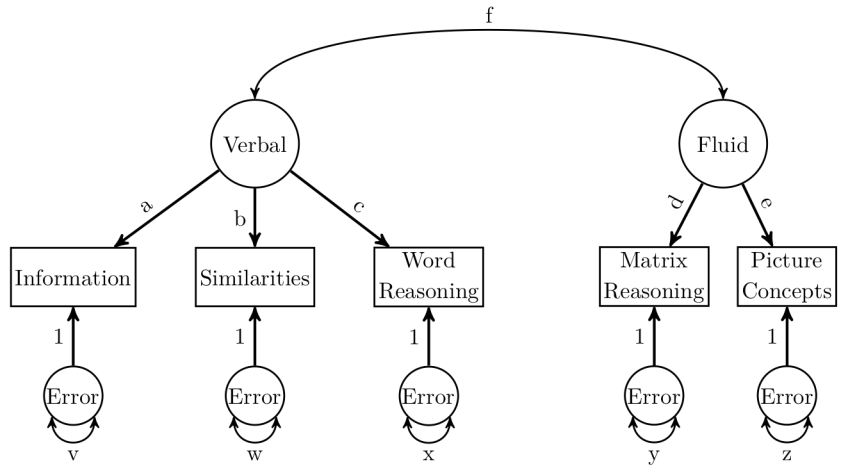


Figure 3.9 Model of five subtests from the Wechsler Intelligence Scale for Children-Fourth Edition with two latent variables.

Table 3.3 Standardized Factor Pattern, Structure, and Correlation Coefficients for the Latent Variable Model in Figure 3.9.

	Factor Pattern		Factor Structure		
Variable	V	F	V	F	Communality
Information	0.86	0.00	0.86	0.71	0.74
Similarities	0.84	0.00	0.84	0.71	0.71
Word Reasoning	0.74	0.00	0.74	0.61	0.55
Matrix Reasoning	0.00	0.69	0.57	0.69	0.47
Picture Concepts	0.00	0.55	0.45	0.55	0.30

Factor Correlations		
	V	F
V	1.00	0.82
F	0.82	1.00

V: Verbal Comprehension; F: Fluid Reasoning.

Índices de Ajuste

ESTIMATION OF CFA MODEL PARAMETERS

The objective of CFA is to obtain estimates for each parameter of the measurement model (i.e., factor loadings, factor variances and covariances, indicator error variances and possibly error covariances) to produce a predicted variance–covariance matrix (symbolized as Σ) that resembles the sample variance–covariance matrix (symbolized as S) as closely as possible. For instance, in overidentified models (such as Figure 3.7A), perfect fit is rarely achieved (i.e., $\Sigma \neq S$). Thus, in the case of a CFA model such as Figure 3.7A, the goal of the analysis is to find a set of factor loadings (λ_{x11}, λ_{x21}, λ_{x31}, λ_{x41}) that yield a predicted covariance matrix (Σ) that best reproduces the input matrix (S)—for example, to find parameter estimates for λ_{x11} and λ_{x21} such that the predicted correlation between X1 and X2 ($\lambda_{x11}\phi_{11}\lambda_{x21}$) closely approximates the sample correlation of these indicators (σ_{21}) (although in the actual estimation process, this occurs simultaneously for all parameters and implied covariances). This process entails a *fitting function*, a mathematical operation to minimize the difference between Σ and S. By far, the fitting function most widely used in applied CFA research (and SEM in general) is ML. The fitting function that is minimized in ML is

$$F_{ML} = \ln|S| - \ln|\Sigma| + \text{trace}[(S)(\Sigma^{-1})] - p \tag{3.15}$$

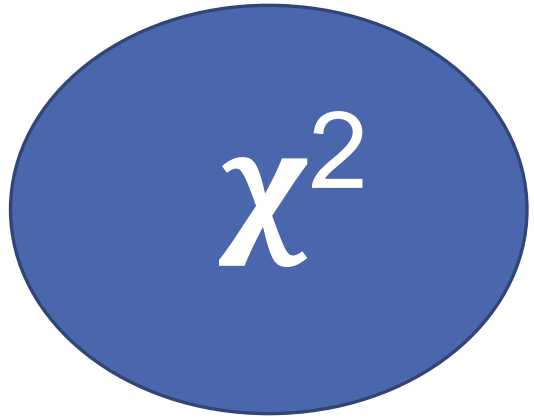
where $|S|$ is the determinant of the input variance–covariance matrix; $|\Sigma|$ is the determinant of the predicted variance–covariance matrix; p is the order of the input matrix (i.e., the number of input indicators); and ln is the natural logarithm. Although a full explication of this function is beyond the scope of this chapter (cf. Bollen, 1989; Eliason, 1993), a few observations are made in effort to foster its conceptual understanding (see also Appendix 3.3).[6] The determinant and trace summarize important information about matrices such as S and Σ. The *determinant* is a single number (i.e., a scalar) that reflects a generalized measure of variance for the entire set of variables contained in the matrix. The *trace* of a matrix is the sum of values on the diagonal (e.g., in a variance–covariance matrix, the trace is the sum of variances). The objective of ML is to minimize the differences between these matrix summaries (i.e., the determinant and trace) for S and Σ. The

DESCRIPTIVE GOODNESS-OF-FIT INDICES

The classic goodness-of-fit index is χ^2. Under typical ML model estimation, χ^2 is calculated as

$$\chi^2 = F_{ML}(N - 1) \tag{3.17}$$

although latent variable software programs (e.g., Mplus, LISREL starting with Version 9.1) increasingly calculate χ^2 by multiplying F_{ML} by N instead of $N - 1$.[8] Using N, the Figure 3.8 model χ^2 is 81.093 (0.4054651 * 200). Because this model is associated with 1 *df*, the critical χ^2 value ($\alpha = .05$) is 3.84 (i.e., $\chi^2 = z^2 = 1.96^2 = 3.8416$). The model χ^2 of 81.093 exceeds the critical value of 3.84, and thus the null hypothesis that $S = \Sigma$ is rejected. Thus a statistically significant χ^2 (latent variable software programs provide



χ^2 test

Asymptotically (i.e., with really large sample sizes), the sample size multiplied by the value of the fit function produces a statistic, T, that follows a χ^2 distribution with the degrees of freedom (df) equal to the amount of non-redundant information minus the number of estimated parameters.[2]

```
# fit value, chi-square value of T, df, and p-value
fitMeasures(wisc4.fit, fit.measures = c("fmin", "chisq", "df", "pvalue"))


##   fmin  chisq     df pvalue
##  0.024 26.166  5.000  0.000
```

If a χ^2 value is "non-significant" (i.e, p-value $> \alpha$), this indicates that the model fits the data relatively well. If the χ^2 value is "significant" (i.e, p-value $< \alpha$), the model does not fit the data well, but it does not necessarily mean the model is not useful, as there are multiple reasons why a χ^2 value might be larger than expected (e.g., sample size, assumption violations).

Some models are variants of other models. That is, they are the same except that one or more parameters are constrained in one model (alternative/more restrictive), but not the other (baseline/less restrictive). Such models are called **nested models**. For example, if I constrained the loadings for the Matrix Reasoning and Picture Concepts to be the same in Figure A.1, then that model would be nested in the model that does not have the constraint. As another example, the hierarchy of invariance models listed in Table 4.1 are nested in each other (the models with larger numbers are nested within the models with smaller numbers). With nested models, the difference in the T statistics' values from the more restrictive to the less restrictive model follows a χ^2 distribution with df equal to the difference in the two models' df. In `lavaan`, two nested models can be compared using the `anova()` function.

Nested Model

The `anova()` function tests a variety of nested models in **R**, not just those from `lavaan`.

`anova()`

```
anova(wisc4.fit, wisc4.2.fit)


## Chi Square Difference Test
##
##             Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## wisc4.fit    5 12689 12732  26.2
## wisc4.2.fit  6 12690 12729  29.4       3.21       1      0.073 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

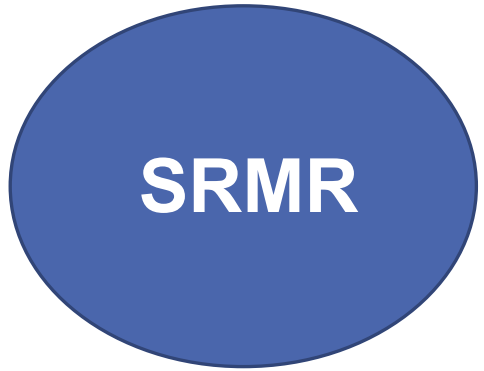
[2] Because of the way the ML fit function is defined in `lavaan`, the T is calculated slightly differently as

$$T = n \times f \times 2 \sim \chi^2$$

Absolute Fit

Absolute fit indices assess model fit at an absolute level; in various ways, they evaluate the reasonability of the hypothesis that $S = \Sigma$ without taking into account other aspects (such as fit in relation to more restricted solutions). Thus χ^2 is an example of an absolute fit index. Another index that falls in this category is the standardized root mean square residual (SRMR). Conceptually, the SRMR can be viewed as the average discrepancy between the *correlations* observed in the input matrix and the correlations predicted by the model (though in actuality the SRMR is a positive square root average; see Eq. 3.18 below). Accordingly, it is derived from a residual correlation matrix (e.g., see Figure 3.8). A similarly named index, the root mean square residual (RMR), reflects the average discrepancy between observed and predicted *covariances*. However, the RMR can be difficult to interpret because its value is affected by the metric of the input variables; thus the SRMR is generally preferred. In most instances (e.g., models involving a single input matrix), the SRMR can be calculated by (1) summing the squared elements of the residual correlation matrix and dividing this sum by the number of elements in this matrix (on and below the diagonal); that is, $a = p(p + 1) / 2$ (Eq. 3.14), and (2) taking the square root of this result. For example, the SRMR of the Figure 3.8 solution would be computed as follows:

$$\text{SRMR} = \text{SQRT}[(0^2 + 0^2 + 0^2 + .4^2 + 0^2 + 0^2)/ 6] = .163 \qquad (3.18)$$



Parsimony Correction

Although sometimes grouped under the category of absolute fit indices (e.g., Hu & Bentler, 1999), *parsimony correction indices* differ from χ^2, SRMR, and so forth by incorporating a penalty function for poor model parsimony (i.e., number of freely estimated parameters as expressed by model *df*). For example, consider a scenario where two different models, Model A and Model B, fit a sample matrix (***S***) equally well at the absolute level; yet the specification of Model B entails more freely estimated parameters than Model A (i.e., Model A has more *df*s than Model B). Indices from the parsimony class would thus favor Model A over Model B because the Model A solution fit the sample data with fewer freely estimated parameters.

A widely used and recommended index from this category is the root mean square error of approximation (RMSEA; Steiger & Lind, 1980). The RMSEA is a population-based index that relies on the *noncentral* χ^2 distribution, which is the distribution of the fitting function (e.g., F_{ML}) when the fit of the model is not perfect. The noncentral χ^2 distribution includes a *noncentrality parameter* (NCP), which expresses the degree of model misspecification. The NCP is estimated as $\chi^2 - df$ (if the result is a negative number, NCP = 0). When the fit of a model is perfect, NCP = 0 and a central χ^2 distribution holds. When the fit of the model is not perfect, the NCP is greater than 0 and shifts the expected value of the distribution to the right of that of the corresponding central χ^2 (cf. Figure 1 in MacCallum, Browne, & Sugawara, 1996). The RMSEA is an "error of approximation" index because it assesses the extent to which a model fits *reasonably* well in the population (as opposed to testing whether the model holds exactly in the population; cf. χ^2). To foster the conceptual basis of the calculation of RMSEA, the NCP is rescaled to the quantity *d*: $d = \chi^2 - df / (N)$. The RMSEA is then computed:

$$\text{RMSEA} = \text{SQRT}(d / df) \tag{3.19}$$

where *df* is the model *df* (although slight variations exist in some programs; e.g., some programs use $N - 1$ instead of N). As can be seen in Eq. 3.19, the RMSEA compensates for the effect of model complexity by conveying discrepancy in fit (*d*) per each *df* in the model. Thus it is sensitive to the number of model parameters; being a population-based index, the RMSEA is relatively insensitive to sample size. The RMSEA from the Figure 3.8 solution would be

$$\text{RMSEA} = \text{SQRT}(.40 / 1) = 0.63$$

where $d = (81.093 - 1) / 200 = 0.40$.

Comparative Fit

Comparative fit indices (also referred to as *incremental fit indices*; e.g., Hu & Bentler, 1998) evaluate the fit of a user-specified solution in relation to a more restricted, nested baseline model. Typically, this baseline model is a "null" or "independence" model in which the covariances among all input indicators are fixed to zero, although no such constraints are placed on the indicator variances (however, other types of null models can and sometimes should be considered; cf. O'Boyle & Williams, 2011). As one might expect, given the relatively liberal criterion of evaluating model fit against a solution positing no relationships among the variables, comparative fit indices often look more favorable (i.e., more suggestive of acceptable model fit) than indices from the preceding categories. Nevertheless, some indices from this category have been found to be among the best behaved of the host of indices that have been introduced in the literature.

One of these indices, the comparative fit index (CFI; Bentler, 1990), is computed as follows:

$$\text{CFI} = 1 - \max[(\chi^2_T - df_T), 0] \,/\, \max[(\chi^2_T - df_T), (\chi^2_B - df_B), 0] \qquad (3.20)$$

where χ^2_T is the χ^2 value of the target model (i.e., the model under evaluation); df_T is the *df* of the target model; χ^2_B is the χ^2 value of the baseline model (i.e., the "null" model); and df_B is the *df* of the baseline model. Also, max indicates to use the largest value—for example, for the numerator, use $(\chi^2_T - df_T)$ or 0, whichever is larger. The χ^2_B and df_B of the null model are included as default output in most software programs. If the user wishes to obtain these values in programs that do provide this information, χ^2_B and df_B can be calculated by fixing all relationships to 0 (but freely estimating the indicator variances). The CFI has a range of possible values between zero and one, with values closer to one implying good model fit. Like the RMSEA, the CFI is based on the NCP (i.e., $\lambda = \chi^2_T - df_T$, included in standard output of some programs such as LISREL), meaning that it uses information from expected values of χ^2_T or χ^2_B (or both, in the case of the CFI) under the noncentral χ^2 distribution associated with $S \neq \Sigma$ (e.g., central χ^2 is a special case of the noncentral χ^2 distribution when $\lambda = 0$). Using the results of the Figure 3.8 model, the CFI would be

$$\text{CFI} = 1 - [(81.093 - 1) \,/\, (227.887 - 3)] = .644$$

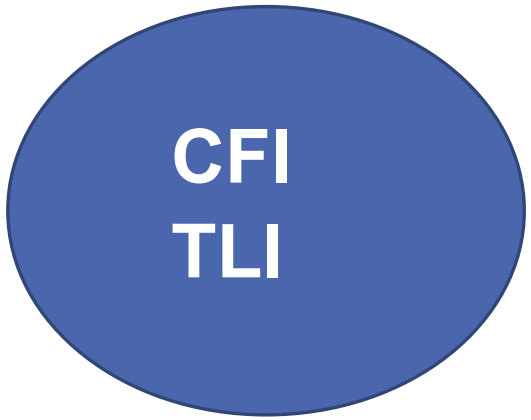
Another popular and generally well-behaved index falling under this category is the Tucker–Lewis index (TLI; Tucker & Lewis, 1973), referred to as the *non-normed fit index* in some programs). In addition, the TLI has features that compensate for the effect of model complexity; that is, as does the RMSEA, the TLI includes a penalty function for adding freely estimated parameters that do not markedly improve the fit of the model. The TLI is calculated by the following formula:

$$\text{TLI} = [(\chi^2_B \,/\, df_B) - (\chi^2_T \,/\, df_T)] \,/\, [(\chi^2_B \,/\, df_B) - 1] \qquad (3.21)$$

where, as with the CFI, χ^2_T is the χ^2 value of the target model (i.e., the model under evaluation); df_T is the *df* of the target model; χ^2_B is the χ^2 value of the baseline model (i.e., the "null" model); and df_B is the *df* of the baseline model. Unlike the CFI, the TLI is non-normed, which means that its values can fall outside the range of zero to one. However, it is interpreted in a fashion similar to the CFI, in that values approaching one are interpreted in accord with good model fit. The TLI for the Figure 3.8 solution is

$$\text{TLI} = [(227.877 \,/\, 3) - (81.093 \,/\, 1)] \,/\, [(227.877 \,/\, 3) - 1] = -0.068$$

The goodness-of-fit indices from each category point to the poor fit of the Figure 3.8 solution: $\chi^2(1) = 81.093$, $p < .001$, SRMR = .163, RMSEA = 0.633, CFI = .644, TLI = –0.068. Although straightforward in the Figure 3.8 example, the issues and guidelines for using these descriptive indices of overall model fit are considered more fully in the next section of this chapter.



General guidelines

- SRMR <=.08
- RMSEA <=.06 (.08)
- CFI e TLI >=.95 (.90)

should lead the researcher to strongly suspect (reject) the solution, CFI and TLI values in the range of .90 and .95 may be indicative of acceptable model fit (e.g., Bentler, 1990).

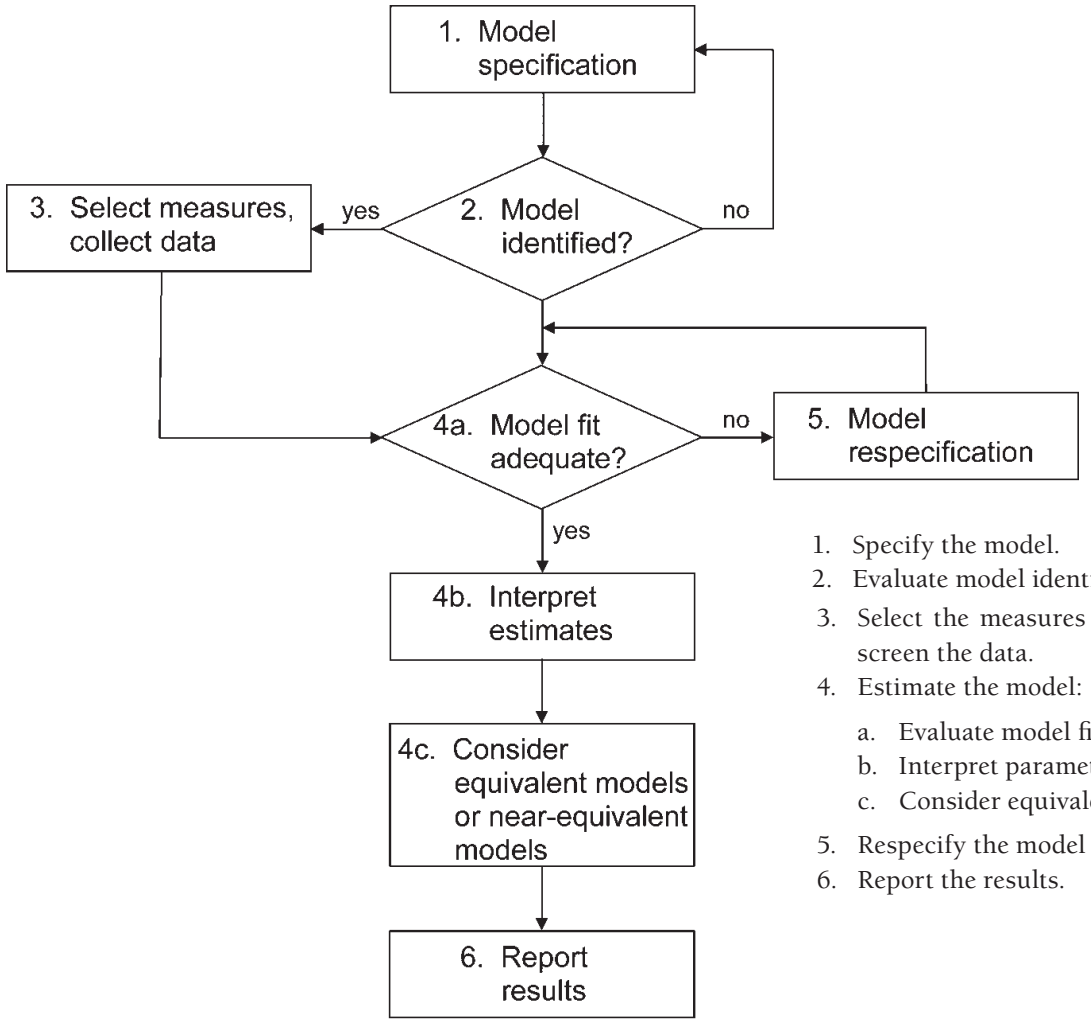
In keeping with the notion that this is a contentious area of methodological research, some researchers have asserted that the Hu and Bentler (1999) guidelines are far too conservative for many types of models, including CFA models consisting of many indicators and several factors where the majority of cross-loadings and error covariances are fixed to zero (cf. Marsh, Hau, & Wen, 2004). Moreover, because the performance of fit statistics and their associated cutoffs have been shown to vary as a function of various aspects of the model (e.g., degree of misspecification, size of factor loadings, number of factors; e.g., Beauducel & Wittman, 2005), the fit statistic thresholds suggested by simulation studies may have limited generalizability to many types of measurement models in applied research.

Nonetheless, when fit indices fall into these "marginal" ranges, it is especially important to consider the consistency of model fit as expressed by the various types of fit indices in tandem with the particular aspects of the analytic situation (e.g., when *N* is somewhat small, an RMSEA = 0.08 may be of less concern if all other indices are strongly in a range suggesting "good" model fit). Again, this underscores the importance of considering fit indices from multiple fit categories (absolute fit, parsimony correction, comparative fit) in tandem with examining other relevant aspects of the solution (e.g., localized areas of ill fit; interpretability and size of parameter estimates). These aspects of model evaluation are discussed in Chapter 4.

Table A.1 Fit Measures Available in `lavaan`.

Fit Measure	Full Name
	Test Statistic and Related
`fmin`	Fit function value
`chisq`	χ^2 value based on the fit function
`df`	Degrees of freedom (df) for model
`pvalue`	p-value for obtained χ^2 value and df
`baseline.chisq`	χ^2 value for baseline model
`baseline.df`	Degrees of freedom for baseline model
`baseline.pvalue`	p-value for the baseline model
`logl`	Logarithm of the likelihood statistic
`unrestricted.logl`	Logarithm of the likelihood statistic for baseline model
`npar`	Number of estimated parameters in the model
`ntotal`	Total sample size
	Alternative Fit Indexes (alphabetically organized)
`agfi`	Adjusted Goodness-of-Fit Index
`aic`	Akaike Information Criterion
`bic`	Bayesian Information Criterion
`bic2`	Bayesian Information Criterion Adjusted for Sample Size
`cfi`	Comparative Fit Index
`cn_05`	Critical n for $\alpha = 0.05$
`cn_01`	Critical n for $\alpha = 0.01$
`ecvi`	Expected Cross-Validation Index
`gfi`	Goodness-of-Fit Index
`ifi`	Incremental Fit Index
`mfi`	McDonald Fit Index
`nfi`	Normed Fit Index
`nnfi`	Non-Normed Fit Index
`pgfi`	Parsimony Goodness-of-Fit Index
`pnfi`	Parsimony Normed Fit Index
`rfi`	Relative Fit Index
`rni`	Relative Noncentrality Index
`rmsea`	Root Mean Square Error of Approximation (RMSEA)
`rmsea.ci.lower`	Lower bound of 95% confidence interval for RMSEA
`rmsea.ci.upper`	Upper bound of 95% confidence interval for RMSEA
`rmsea.pvalue`	p-value associated with H_0: RMSEA < 0.05
`rmr`	Root Mean Square Residual (includes means, if used in model)
`rmr_nomean`	Root Mean Square Residual (no means)
`srmr`	Standardized Root Mean Square Residual (includes means, if used in model)
`srmr_nomean`	Standardized Root Mean Square Residual (no means)
`tli`	Tucker-Lewis Index

n: sample size. α: Type 1 error. H_0: Null hypothesis.



1. Specify the model.
2. Evaluate model identification (if not identified, go back to step 1).
3. Select the measures (operationalize the constructs) and collect, prepare, and screen the data.
4. Estimate the model:
   a. Evaluate model fit (if poor, skip to step 5).
   b. Interpret parameter estimates.
   c. Consider equivalent or near-equivalent models (skip to step 6).
5. Respecify the model (return to step 4).
6. Report the results.

FIGURE 5.1. Flowchart of the basic steps of SEM.

3.6 Writing the Results

Believe it or not, there is actually literature on writing LVM literature (e.g., Boomsma, Hoyle, & Panter, 2012; McDonald & Ho, 2002). While there are some differences in the authors' suggestions, there are also many areas of consensus. They all agree that a LVM should include:

1. A theoretical and empirical justification for the hypothesized model;
2. A complete description of how the LVMs were specified (i.e., the indicator variables for each LV, the scaling of the LVs, a description of what parameters were estimated and constrained);
3. A description of sample (i.e., demographic information, sample size, sampling method);
4. A description of the type of data used (e.g., nominal, continuous) and descriptive statistics;
5. Tests of assumptions (specifically that the indicator variables follow a multivariate normal distribution and estimator used (see Appendix A);
6. A description of missing data and how the missing data was handled (see Chapter 7);
7. The software and version used to fit the model;
8. Measures, and the criteria used, to judge model fit (see Appendix A); and
9. Any alterations made to the original model based on model fit or modification indices.

In addition, all reports should have a table with all parameter estimates (i.e., loadings, error variances, latent [co]variances), their standard errors, and standardized versions for the final model as well as any other significant model if more than one model was fit to the data. In Section 2.6, I discussed the `xtable()` function, which works on `lavaan` output from the `parameterEstimates()` function, thus creating an easy way to export `lavaan` parameter results into a table. Likewise, to increase research replicability, the LVM manuscript should also include the sample covariance matrix(es) either in the body of the text or in an appendix. The `xtable()` function can help facilitate this process as well.

While not mandatory, it is very useful to include a neatly drawn path model of the final LVM, as well as any other significant model if more than one model was fit to the data. In the diagram, place the standardized or unstandardized (or both) coefficients along a given path. If using unstandardized coefficients, or leaving out any part of the LVM for ease of reading (e.g., the error variance terms), indicate this in the figure's caption. Nicol and Pexman (2010) give some examples of publication-ready diagrams that include LVs.