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Fluid intelligence as a predictor of learning: A longitudinal multilevel approach applied to math ☆

Ricardo Primi [a,*], Maria Eugénia Ferrão [b], Leandro S. Almeida [c]

[a] University of São Francisco, Brazil
[b] Beira Interior University, Portugal
[c] University of Minho, Portugal

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ABSTRACT

The association between fluid intelligence and inter-individual differences was investigated using multilevel growth curve modeling applied to data measuring intra-individual improvement on math achievement tests. A sample of 166 students (88 boys and 78 girls), ranging in age from 11 to 14 (M = 12.3, SD = 0.64), was tested. These individuals took four math achievement tests, which were vertically equated via Item Response Theory, at the beginning and end of the seventh and eighth grade. The cognitive abilities studied were Numerical Reasoning, Abstract Reasoning, Verbal Reasoning, and Spatial Reasoning (as measured by the Differential Reasoning Test). The general cognitive factor was significantly associated with the parameters of initial level (intercept) and rate of change (slope). A high level of intelligence was associated with higher initial scores, as well as a steeper rise in math scores across the two years.

In the psychometric tradition, fluid intelligence (*Gf*) is defined as the use of deliberate mental operations to solve novel problems (i.e., tasks that cannot be performed as a function of simple memorization or routine). Such mental operations include drawing inferences, concept formation, classification, generating and testing hypothesis, identifying relations, comprehending implications, problem solving, extrapolating, and transforming information (McGrew, 2009; McGrew & Evans, 2004; Kane & Gray, 2005). Fluid intelligence is contrasted with crystallized intelligence (Gc), which refers to the wealth (breadth and depth) of acquired knowledge (Cattell, 1963, 1971; Horn, 1991). Ackerman (1996) also refers to two kinds of general capacities, intelligence as process versus intelligence as knowledge, both involved in cognitive functioning.

Fluid intelligence is closely related to general or *g*-factor intelligence (Ackerman, Beier & Boyle, 2002; Blair, 2006; Salthhouse, Pink & Tucker-Drob, 2008), which is itself based in executive functions related to perception, attention and working memory (Ackerman, Beier & Boyle, 2005; Engle, Tuholski, Laughlin & Conway, 1999; D'Esposito, 2007; Kane, Habrick & Conway, 2005; Shimamura, 2000; Smith & Jonides, 1999). Fluid intelligence is also recognized as a causal factor in learning, especially in novel situations (Kvist & Gustafsson, 2008; Voelkle, Wittmann, & Ackerman, 2006; Watkins, Lei & Canivez, 2007). Although fluid and crystallized intelligences are viewed as differentiated constructs, *Gf* provides the foundation for Gc since it supports the acquisition of skills and knowledge that is the essence of Gc, as proposed by Cattell's investment theory (Cattell,1971). In this sense, *Gf* is also conceived of as the ability to learn new information and, consequently, to adapt to novel situations. This occurs especially in the early phases of learning, when the learner encounters new information and new experiences that are initially perceived as being somewhat disorganized and disconnected. In those situations, the ability to work in a systematic and controlled manner, with the goal of finding regularities in information, is a key strategy for the creation of stable representations and the formation of new knowledge (McArdle & Hamagami, 2006; McArdle, Hamagami, Meredith, & Bradway, 2000). Novel and complex situations require higher cognitive abilities for the systematic processes of selection, maintenance, updating, and rerouting, which are crucial for dealing with situations of "information overload" (Primi, 2002; Primi et al., 2001). Controlled learning studies which use laboratory tasks to measure rate of learning via repeated measures (Ackerman & Cianciolo, 2002; Voelkle et al., 2006), as well as those attempting to ascertain the structural relationships between abilities and learning (Snow, Kyllonen, & Marshalek, 1984), have shown that the strongest correlations between fluid intelligence (*Gf*) and learning are found when tasks are both new and complex. Thus, novelty and complexity of information moderate the correlation between fluid intelligence and learning.

Several studies have shown that fluid intelligence is an important predictor of math achievement (Floyd, Evans, & McGrew, 2003; McGrew, 2008; McGrew & Hessler, 1995; Taub, Floyd, Keith, &

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* Corresponding author. Rua Ferreira Penteado, 1518, Apt. 41, Bairro Cambuí, Campinas, São Paulo, CEP 13025-357, Brazil. Tel.: +55 19 81492244.
E-mail address: rprimi@mac.com (R. Primi).

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McGrew, 2008). The understanding of math concepts requires the formation of abstract representations of quantitative and qualitative relations between variables. Further, it requires the ability to link second order relationships in a logical and ordered manner and the ability to manipulate visual representations. Thus, owing to the inherent complexity of mathematics instruction, we hypothesize that success in learning math requires, and is therefore correlated with, fluid intelligence (Busse, Berninger, Smith, & Hildebrand, 2001; Geary, 1993, 2007).

Even though a fundamental aspect of fluid intelligence is the ability to learn in novel situations, there is some debate about this definition in the literature, inconsistent data have been presented on the relationship between fluid intelligence and measures of learning. It has been shown that intelligence is associated with initial level but not with rate of improvement on simple learning tasks (Lohman, 1999; Woodrow, 1946; Zhang, Davis, Salthouse, & Tucker-Drob, 2007; Tamez, Myerson, & Hale, 2008; Williams & Pearlberg, 2006). Moreover, psychometric difficulties and misunderstandings often arise from the use of difference scores (post-test minus pre-test scores) as a measure of learning (Rogosa & Willett, 1983; Willett, 1989, 1997). Recent applications of multilevel modeling (Bryk & Raudenbush, 1987; Plewis, 2005) and latent growth curve analysis (McArdle & Hamagami, 2001) have been proposed as a way of overcoming problems in this regard.

As was pointed out by Voelkle et al. (2006), one important difficulty in the study of the relationship between ability and learning is the determination of actual acquisition of a specific skill or concept. Usually studies use a measure of learning at a single point in time as the criterion to be predicted. But in order to measure actual acquisition, it is necessary to have longitudinal or repeated measures data (viz., two or more within-subject measures over time). It is also necessary to use adequate statistical models such as those that take into account the hierarchical structure of data (e.g., repeated measures grouped within students). Multilevel models (Goldstein, 2003; Bryk & Raudenbush, 1987; McArdle & Hamagami, 2001; Plewis, 2005), specifically the growth curve model, are now widely used to accommodate such a data structure. Recent studies have applied these methods to the investigation of the association between cognitive abilities and rates of learning (i.e., Swanson, Jerman, & Zheng, 2008; Voelkle et al., 2006; Zhang et al., 2007). With the exception of Swanson et al. (2008), we are not aware of any other research using growth curve modeling to test the association between fluid intelligence and math learning. Thus, although fluid intelligence is theoretically considered to be an influential factor for complex learning such as math, there is little empirical evidence of its association with actual measures of learning based on longitudinal growth curve analysis.

This paper contributes to that matter by pursuing two objectives. The first is to contribute to the field by having gathered empirical evidence about the relation between fluid intelligence and individual differences in improved math achievement test scores. Thus, we aimed to test the hypothesis that fluid intelligence is not only an important predictor of math achievement (which is also related to past learning) at the concurrent or entry level for the longitudinal measures, but that it is also a predictor of growth. The other purpose of this paper was to illustrate multilevel modeling using longitudinal data in the context of intelligence–learning interaction research.

1. Method

1.1. Participants

The data for this study comes from a larger database of a school effectiveness research project (3EM), coordinated by the second author (Ferrão, 2009; Ferrão & Goldstein, 2009). The population is defined by students enrolled in compulsory education in the region of Cova da Beira, a NUT III Portuguese region. The survey design is longitudinal. Data were collected at the beginning and at the end of academic years 2005/6, 2006/7 and 2007/8. Two cohorts of students were considered. In 2005/6 the 1st, 3rd, 5th, 7th and 8th grade students were involved. They were monitored in the 2nd, 4th, 6th, 8th and 9th grades, respectively, and a new cohort at the 1st, 3rd, 5th, and 7th years was surveyed. In 2007/8 all these students were monitored again. The random sample is representative at the level of county and NUT III region (Vicente, 2007).

For the purposes of this paper, we focused on the students that began the 7th grade in 2005/6 and ended the 8th in 2006/7. Among the 166 pupils, 88 were boys and 78 girls. Ages varied from 11 to 14 ($M = 12.3$, $SD = 0.64$) at the beginning of the study. The choice of 7th grade students is related to the fact that in Portugal the transition between elementary and lower education is marked by high rates of student repetition (no promotion to the next grade).

1.2. Materials

1.2.1. Math tests

3EMat is a battery of tests designed for the assessment of Math skills throughout primary, elementary and lower secondary education (Ferrão et al., 2005). Each test includes around 30 selected items covering the core curriculum for each grade. Item calibration (discrimination and difficulty) was done during the pre-test at the end of 2004/5. A two-parameter item response logistic model (Birnbaum, 1968), implemented by BILOG computer software for the estimation of item and ability parameters (Zimowski, Muraki, Mislevy, & Bock 1996), was used. The Bayes Expected a Posteriori (EAP) procedure with a latent scale (normal standard) was applied. The test booklets included common items (about 30%) from adjacent grades in order to allow posterior vertical equating. The distribution of items per subjects is approximately as follows: 7th grade, Geometry 24%; Numbers 36%; Equations 27%; and Statistics 13%; 8th grade, Geometry 39%; Numbers 30%; Equations 12%; Functions 13%; and Statistics 6%.

1.2.2. Intelligence tests

Cognitive abilities were assessed using the Differential Reasoning Tests Battery (Almeida, 1988; Almeida, 1992). Although tests are based on analogy or series tasks combining different contents, the same cognitive operation–reasoning or fluid intelligence–is evaluated for each of the different domains: Numerical Reasoning (NR), consisting of 30 numerical series items involving simple arithmetic operations; Abstract Reasoning (AR) consisting of 40 involving abstract analogies of geometric figures; Verbal Reasoning (VR) consisting of 40 items involving verbal analogies; and Spatial Reasoning (SR) consisting of 30 spatial series related to the rotation of the six faces of a cube.

The Kuder–Richardson coefficient for internal consistency varies from 0.78 for VR to 0.91 for NR. Factor analysis revealed a single general factor (near 60% of the variance explained). This is considered to represent *Gf*. The NR, AR, VR and SR scores are the residuals of the linear regression of *Gf* on each raw score, respectively. The analysis includes data collected at the beginning of the study.

1.2.3. Statistical model

The growth curve multilevel model was used in order to estimate individual growth parameters, to check whether the variance of these parameters across individuals was statistically significant, and to investigate their association with predictive variables. Level 1 consists of repeated observations hierarchically nested within pupils to constitute Level 2. Individual growth trajectories are modeled at Level 1 by Eq. (1), where parameters are considered to be random across pupils. At Level 2, intelligence variables can be tested for their capacity to predict personal outcome variables; that is, π_{0i} math achievement at the beginning of the study and π_{1i} the average of change in one year. The model is defined by Eqs. (1), (2) and assumptions (3):

Level 1 equation

$$Y_{ti} = \pi_{0i} + \pi_{1i}a_{ti} + e_{ti} \quad (1)$$

Level 2 equations

$$\pi_{0i} = \beta_{00} + \sum_{q=1}^{Q} \beta_{0q}X_{qi} + r_{0i}$$
$$\pi_{1i} = \beta_{10} + \sum_{q=1}^{Q} \beta_{1q}X_{qi} + r_{1i} \quad (2)$$

Assumptions

$$\begin{bmatrix} r_{0i} \\ r_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{r0} & \\ \sigma_{r01} & \sigma^2_{r1} \end{bmatrix}\right) \quad (3)$$
$$e_{ti} \sim N(0, \sigma^2_e),$$

where Y_{ti} is the dependent variable (math achievement) of student i at time t; π_{0i} represents the math achievement of student i when a_{ti} is equal to 0 (0: beginning of 7th grade; 1: end of 7th grade; 2: end of 8th grade); π_{1i}, the growth rate for student i over a year. The term e_{ti} is the level-1 within-pupil residual. This term is assumed to be independent and normally distributed, with mean 0 and variance σ^2_e. β_{00} is the overall mean of math achievement at the beginning of 7th grade;$\beta01$ is the average growth in mathematics achievement across students. Both parameters are conditioned on q predictor variables (X_{qi}), such as *Gf*, NR, etc.; r_{0i} is the deviation of student i from the mean initial status and r_{1i} the deviation of student i from the average growth on math achievement (again, conditioned by q predictors). These terms are assumed to be normally distributed, with mean zero and variances σ^2_{r0}, σ^2_{r1}, respectively, and the covariance between those terms is σ_{r01}. The β_{0q}represents the relationship between intelligence variables and initial math achievement, while β_{1q} represents the association between such variables and growth.

According to the working hypotheses β_{0q} would differ significantly from zero, since *Gf* is associated with math achievement. If *Gf* indeed captures some underlying reasoning mechanism important for math learning, high *Gf* students would be expected to reveal greater growth, as determined by a comparison of their change from prior achievement to that of an average *Gf* student. Hence we expected that β_{1q} would also differ from zero, and if this was the case, we would argue that this is evidence in favor of the influential role of *Gf* on math learning.

2. Results

2.1. Descriptive statistics

Table 1 presents the descriptive statistics for all variables used. It can be seen that achievement in mathematics tends to increase from the first to the second occasion, but even more from the third to the fourth occasion. Another pattern is that math achievement at the end of the year is peaked and varies more (see positive kurtosis) than at the beginning of the year. Intelligence variables are considered in the adequate range with the exception of NR, which seems to be more difficult for this sample of students.

Fig. 1 presents the individual growth curves for the 166 subjects divided by three groups based on *Gf* raw scores quartiles (below 25 percentile, between 25 and 75 and above 75 percentile) suggesting that there is ample inter-individual variability in patterns of intra-individual growth and this pattern appears to be related to intelligence.

Table 2 presents the correlations between all variables and it can be observed that all of them are positively correlated. This evidence corroborates past research results (Almeida, 1992; Primi & Almeida, 2000). It is interesting to note that difference scores correlated significantly with each other and with intelligence measures, showing that there was reliable inter-individual differences in rate of learning associated with fluid intelligence. The negative correlation between math3 and math4 −math3 suggest a possible ceiling effect for high abilities students restricting the amount of gain illustrating the difficulties that surrounds measures of learning.

Table 1
Descriptive statistics of math (criterion) and intelligence (predictor variables).

	Min.	Max.	Mean	Std. dev.	Skew.	Kurt.
Measurements						
Math1	−2.28	2.30	0.21	0.98	−0.16	−0.36
Math2	−2.56	5.63	0.70	1.49	0.68	0.74
Math2 − math1	−1.63	3.64	0.48	0.98	0.73	0.99
Math3	−4.09	4.98	0.70	1.49	0.05	0.40
Math4	−3.15	5.84	1.52	1.49	0.33	1.26
Math4 − math3	−3.92	3.94	0.82	1.37	−0.41	0.29
NR	0.03	0.40	0.20	0.08	−0.04	−0.67
VR	0.18	0.73	0.45	0.13	0.00	−0.77
SR	0.10	0.87	0.41	0.16	0.15	−0.44
AR	0.09	0.86	0.55	0.15	−0.71	0.06
BPRD	0.16	0.65	0.40	0.10	−0.10	−0.67

Note. Math1: math performance at the beginning of academic year 2005/6 (occasion 1); math2: math performance at the end of academic year 2005/6 (occasion 2); math3: math performance at the beginning of academic year 2006/7 (occasion 3); math4: math performance at the end of academic year 2006/7 (occasion 4); math2 − math1: simple difference score subtracting math1 from math2; math4 − math3: simple difference score subtracting math3 from math4; NR: Numerical Reasoning; VR: Verbal Reasoning; SR: Spatial Reasoning; AR: Abstract Reasoning; BPRD: total score (general factor) on the four subtests.

2.2. Statistical modeling

Table 3 presents the estimates of the Linear Growth Model parameters, which were obtained by an Iterative Generalized Least Squares algorithm implemented in MLWIN (Rasbash, Steele, William, & Prosser, 2005) based on 498 cases (3 time occasions for 166 students). Model 0 (null model) is comprised of Eqs (1) and (2) without predictors. Approximately equal amounts of variance were observed on math achievement among individuals and on individual growth during the two-year period. Two questions are addressed by model 0: it tells us what the average of change is (equal or different from zero); and whether there is evidence of inter-individual variation in individual growth. Model 1a includes the a_{ti} math achievement, and considers that initial achievement varies across students ($\beta_{00}$and σ^2_{00}), but growth

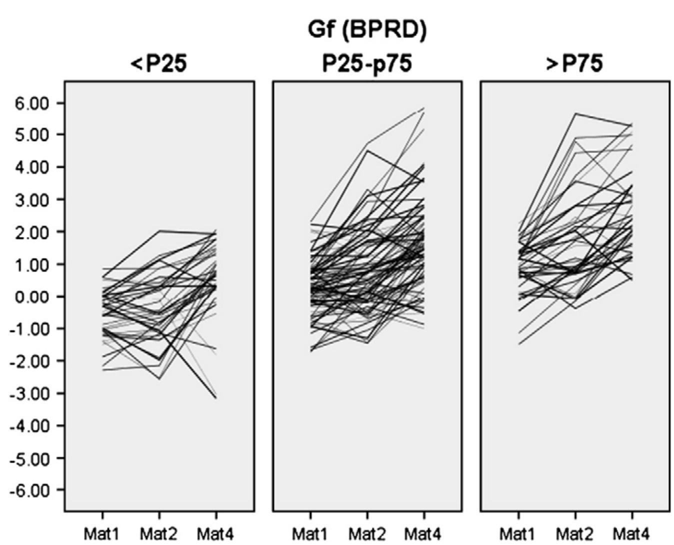

Fig. 1. Individual growth curves in math of three subgroups differing in fluid intelligence.

Table 2
Correlation among variables of the study.

	1	2	3	4	5	6	7	8	9	10	11
1. math1	1										
2. math2	0.76**	1									
3. math2 − math1	0.16*	0.76**	1								
4. math3	0.53**	0.54**	0.30**	1							
5. math4	0.61**	0.73**	0.51**	0.58**	1						
6. math4 − math3	0.26**	0.39**	0.34**	−0.21**	0.68**	1					
7. NR	0.59**	0.56**	0.27**	0.36**	0.55**	0.34**	1				
8. VR	0.44**	0.45**	0.25**	0.21**	0.39**	0.28**	0.53**	1			
9. SR	0.43**	0.34**	0.09	0.29**	0.31**	0.11	0.49**	0.42**	1		
10. AR	0.53**	0.50**	0.23**	0.24**	0.46**	0.34**	0.55**	0.52**	0.45**	1	
11. BPRD	0.63**	0.59**	0.27**	0.35**	0.55**	0.34**	0.82**	0.79**	0.75**	0.80**	1

*$p<0.05$; **$p<0.01$.

(β_{10}) is fixed. These results, presented in the middle part of Table 3, show that the mean growth rate is indeed statistically different from zero ($\beta_{10}=0.567$, $t=12.6$, $p<0.01$), indicating that a unit change in time (corresponding to a single year) is, on the average, associated with a 0.57 increase in math achievement. Moreover, the initial achievement reveals a considerable amount of variance across students ($\sigma^2_{00}=0.945$).

Model 1b is an extension of Model 1a in that it allows the growth parameter to vary randomly across students. In this way, it is possible to test the second basic question of whether this modification propitiates a better-fitted model, therefore being suggestive of the existence of inter-individual differences in individual growth. The deviance test for the goodness of fit suggests that both parameters are statistically significant. The variance of growth is $\sigma^2_{01}=0.285$, which is statistically significant, but lower than the variance involved in the initial achievement. This model suggests that the correlation between initial achievement and growth is 0.120.

Table 4 present the results for Model 2, which includes intelligence as predictor of initial status (r_{0i}) and growth (r_{1i}), after testing all ten possible combinations. Therefore, the results of the best fitted model are shown (only the significant predictors). The deviance (as compared with model 1b) is 106.19 ($df=3$) which indicates that the inclusion of these predictors generally reduces the discrepancies between observed and predicted math scores.

Estimates suggest a strong relationship between intelligence scores (*Gf* and NR) and initial math achievement. More importantly, *Gf* also served as a significant predictor of the growth rate. Fig. 1 shows individual growth curves for the 166 subjects, separated into three groups of increasing levels of fluid intelligence. It can be seen that the slope is slightly less steep for subjects in the low fluid group (left panel figure). A comparison of variances of the initial status and growth rate in Model 1b with variances in Model 3, allowed for calculation of the amount of variance accounted for by intelligence predictors. This was done by comparing the difference in total variance (estimated by the unconditional model, 0.945 and 0.285, respectively, for initial achievement and growth rate) and the residual variance (based on the fitted model including predictors, 0.556 and 0.259) relative to the total variance (Raudenbush & Bryk, 2002). Thus for the initial status, 0.41 of the variance ((0.945 − 0.556)/0.945) is accounted for by intelligence tests whereas for growth rate, 0.09 ((0.285 − 0.259)/0.285) is accounted for by the predictors. Thus, the results of this final model provide evidence that fluid intelligence is capable of predicting growth rate above and beyond its capacity to predict math scores (initial status). This is consistent with our central hypothesis regarding the role of fluid intelligence in math learning.

Table 3
Estimated parameters of the multilevel linear growth model for math achievement with predictors not included (unconditional model).

Unconditional linear growth models	Parameter	Coef./Var.	se	*t* ratio
Model 0: Baseline Model and variance components estimation				
Level 2 variance	$\text{Var}(r_{0i})=\sigma^2_{00}$	1.062	0.156	
Level 1 variance	$\text{Var}(e_{ti})=\sigma^2_{0ti}$	1.021	0.079	
Deviance	−2*Loglikelihood	1658.63		
Model 1a: Including moment predictor and its coefficients as fixed parameters				
Fixed effects				
Mean initial math achievement	β_{00}	0.215	0.075	2.866
Mean math growth rate	β_{10}	0.567	0.045	12.600
Random effect				
Initial math achievement	$\text{Var}(r_{0i})=\sigma^2_{00}$	0.945	0.104	
Error (Level 1 residual variance)	$\text{Var}(e_{ti})=\sigma^2_{1ti}$	0.661	0.077	
	−2*Loglikelihood	1496.78		
Model 1b: Including moment predictor and its coefficients as random parameters varying across subjects (unconditional model)				
Fixed effects				
Mean initial math achievement	β_{00}	0.215	0.075	2.866
Mean math growth rate	β_{10}	0.567	0.053	10.698
Random effect				
Initial math achievement	$\text{Var}(r_{0i})=\sigma^2_{00}$	0.945	0.104	
Growth rate	$\text{Var}(r_{1i})=\sigma^2_{01}$	0.285	0.056	
Covariance between initial achievement and growth rate	$\text{Cov}(r_{0i},r_{1i})$	0.064	0.052	
Error (Level 1 residual variance)	$\text{Var}(e_{ti})=\sigma^2_{1ti}$	0.377	0.041	
Deviance	−2*Loglikelihood	1461.25		
Difference relative to Model 1a ($df=2$)		35.53		

Table 4
Estimated parameters of the multilevel linear growth model for math achievement with predictors included (conditional model).

Conditional linear growth models	Parameter	Coef./Var.	se	*t* ratio
Model 2: Final model including predictors (conditional)				
Fixed effects				
Mean initial math achievement	β_{00}	0.215	0.058	3.706
Mean math growth rate	β_{10}	0.567	0.052	10.903
Predictors for initial math achievement				
Gf	β_{01}	0.616	0.063	9.777
NR	β_{02}	2.054	0.942	2.180
Predictor for growth in math achievement				
Gf	β_{11}	0.274	0.065	4.215
Random effects				
Initial math achievement	$\text{Var}(r_{0i})=\sigma^2_{00}$	0.556	0.061	
Growth rate	$\text{Var}(r_{1i})=\sigma^2_{01}$	0.259	0.052	
Covariance between initial achievement and growth rate	$\text{Cov}(r_{0i},r_{1i})$	−0.057	0.039	
Error (Level 1 residual variance)	$\text{Var}(e_{ti})=\sigma^2_{1ti}$	0.364	0.040	
Deviance	−2*Loglikelihood	1355.06		
Diference as compared with Model 2b ($df=3$)		106.19		

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3. Discussion

The present study investigated the association of fluid intelligence with inter-individual differences in intra-individual growth on math achievement. It has also illustrated the utility of using multilevel modeling in the analysis of longitudinal data in intelligence research. The general results are in accordance with a common finding in the literature that individual differences in fluid intelligence are strongly related to math achievement when the measures are taken concurrently (Floyd et al., 2003; McGrew & Hessler, 1995; Taub et al., 2008). It then shows that there are important inter-individual differences in intra-individual growth patterns in math achievement over a two-year period, with some subjects increasing their math scores at a faster rate than others. One substantial finding was that these individual differences in growth rate could be explained, at least in part, by fluid intelligence. Individuals with higher fluid intelligence reveal a faster increase in math scores over a span of two years than do individuals with a lower fluid intelligence.

This evidence is in accordance with similar findings from previous research using growth curve modeling that encountered a correlation between rate of change (Willett, 1989, 1997) and intelligence factors (Swanson et al., 2008; Voelkle et al., 2006). It is also consistent with other studies, using different methodological approaches, which found a positive correlation between fluid intelligence and rate of learning (Hambrick et al., 2008; Tamez et al., 2008; Watkins et al., 2007; Williams & Pearlberg, 2006). Moreover, it is consistent with the results of the controlled experimental studies of Klauer and Phye (2008) designed to develop fluid abilities and which showed that increases in inductive reasoning abilities were also accompanied by improved learning of classroom subject matter.

The results of this study support the hypothesis that fluid intelligence is an important factor in learning a math curriculum. The general explanation is that fluid intelligence is associated with reasoning abilities (both inductive and deductive) involved in understanding and solving novel problems (Ackerman, & Cianciolo, 2002; Blair, 2006; Busse et al., 2001; Geary, 1993, 2007; Heitz et al., 2005; Kane et al., 2005; Primi, 2002; Snow et al., 1984; Swanson et al., 2008). However, the results are partly inconsistent with those of Zhang et al. (2007). These latter authors applied latent growth curve modeling to analyze a laboratory memory task involving verbal and spatial stimulus and found no general association between rate of learning (slope parameter) and measures of fluid and crystallized intelligence. They only found that these measures were correlated with the intercept, i.e., the concurrent initial levels. The only exception was for a younger sample where their results are comparable to ours with respect to slope parameter.

There are many methodological differences that can explain this apparent inconsistency. The most significant of these relates to the dimension of task complexity, which has been found to moderate the relationship between intelligence and learning (Ackerman, 1996; Ackerman et al., 2002; Snow et al., 1984; Voelkle et al., 2006). The learning task in Zhang et al. (2007) study required that the subjects had to memorize unrelated words through repetitive exposure and spatial positions of previously viewed figures in a matrix, a task which may not required much attentional control, processing and recombination of new information, as would have been required for a more complex task such as learn a math concept. Learning parameters of simple tasks would not be expected to correlate with fluid intelligence measures. Perhaps a slightly more complex task, such as those used by Tamez et al. (2008) and Williams and Pearlberg (2006) involving a group of associated words, would have been sufficient to reveal the association with fluid intelligence found in these latter two studies. Conversely, learning measures involving the domain of math taught at school, which are more comparable to the complex tasks used by Ackerman et al. (2002), Voelkle et al. (2006), Snow et al. (1984), and Swanson et al. (2008), would show this relationship and may also explain the similarity of results with these studies.

Other relevant methodological difference include the time lag between measures used to derivate slope parameters that was minutes for Zhang et al. (2007) and one year in the present study. This difference may again suggest that the construct underlying learning measures differs between studies and could explain the apparent inconsistencies. Finally, since results are similar for comparable age groups it could be suggested that age may also moderates the association of intelligence and learning.

In summary fluid intelligence has been shown to be related to faster learning of math consistent with the definition of intelligence as an ability to learn. Hence, as was illustrated in this study, growth curve modeling is a flexible and important methodological tool for the investigation of patterns of learning and its association with predictor variables, and can be very helpful in answering this type of research questions about the underlying mechanism of intelligence–learning relationships.

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